H

National
Qualifications
2024

2024 Mathematics

Higher - Paper 1

Question Paper Finalised Marking Instructions

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Marking Instructions for each question

Question			Generic scheme	Illustrative scheme	Max mark
1.			•[1] use $m=\tan\theta$ •[2] evaluate exact value •[3] determine equation	•[1] $m=\tan 30°$ •[2] $\frac{1}{\sqrt{3}}$ •[3] eg $y=\frac{1}{\sqrt{3}}x+4$ or $\sqrt{3}y-4\sqrt{3}=x$	3

Notes:

1. Do not award •[1] for $m=\tan^{-1}30°$. However •[2] and •[3] are still available.
2. Do not penalise the omission of a degree symbol at •[1].
3. Where candidates make no reference to a trigonometric ratio, or use an incorrect trigonometric ratio, •[1] and •[2] are unavailable. See Candidate A.
4. •[3] is only available as a consequence of attempting to use a tan ratio. See Candidate F.
5. •[3] is not available for using a gradient of 30.
6. At •[3] accept any rearrangement of a candidate's equation where constant terms have been simplified.
7. Accept $y-4=\frac{1}{\sqrt{3}}(x)$ but not $y-4=\frac{1}{\sqrt{3}}(x-0)$ for •[3].

Commonly Observed Responses:

Candidate A - no trig ratio $m=\frac{1}{\sqrt{3}}$ •[1] ^ •[2] ✓[2] $y=\frac{1}{\sqrt{3}}x+4$ •[3] ✓[1]	**Candidate B** $m=\tan\theta$ •[1] ^ $y=\frac{1}{\sqrt{3}}x+4$ •[2] ✓ •[3] ✓	**Candidate C** $m=\tan\theta$ •[1] ^ $y=\sqrt{3}x+4$ •[2] ✗ •[3] ✗
Candidate D $m=\tan\theta=30$ •[1] ✗ $m=\frac{1}{\sqrt{3}}$ •[2] ✓[1] $y=\frac{1}{\sqrt{3}}x+4$ •[3] ✓[1]	**Candidate E - no reference to *m*** $\tan 30°=\frac{1}{\sqrt{3}}$ •[2] ✓ $y-4=\frac{1}{\sqrt{3}}(x-0)$ •[1] ✓ $y=\frac{1}{\sqrt{3}}x+4$ •[3] ✓	**Candidate F - not using tan** $m=\sin 30°$ •[1] ✗ $m=\frac{1}{2}$ •[2] ✓[2] $y=\frac{1}{2}x+4$ •[3] ✓[2]

Question			Generic scheme	Illustrative scheme	Max mark
2.	(a)		•[1] calculate second term	•[1] 16	**1**

Notes:

1. Candidates who use $u_0 = 20$ and then calculate $u_1 = 16$ gain •[1].

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
	(b)	(i)	•[2] communicate condition for limit to exist	•[2] a limit exists as $-1 < \frac{1}{5} < 1$	**1**
		(ii)	•[3] know how to calculate a limit •[4] calculate limit	•[3] $\frac{12}{1-\frac{1}{5}}$ or $L = \frac{1}{5}L + 12$ •[4] 15	**2**

Notes:

2. For •[2] accept:

   any of '$-1 < \frac{1}{5} < 1$' or '$\left|\frac{1}{5}\right| < 1$' or '$0 < \frac{1}{5} < 1$' with no further comment;

   or statements such as:

   '$\frac{1}{5}$ lies between -1 and 1' or '$\frac{1}{5}$ is a proper fraction'.

3. •[2] is not available for:

   '$-1 \le \frac{1}{5} \le 1$' or '$\frac{1}{5} < 1$'

   or statements such as:

   'It is between -1 and 1.' or '$\frac{1}{5}$ is a fraction'.

4. Candidates who state $-1 < a < 1$ can only gain •[2] if it is explicitly stated that $a = \frac{1}{5}$.
5. Do not accept $L = \frac{b}{1-a}$ with no further working for •[3].
6. •[3] and •[4] are not available to candidates who conjecture $L = 15$ following the calculation of further terms in the sequence.
7. For $L = 15$ with no working award 0/2.
8. •[4] is only available where •[3] has been awarded.

Commonly Observed Responses:

Candidate A	**Candidate B - no explicit reference to *a***
$a = \frac{1}{5}$ $-1 < a < 1$ so a limit exists •[2] ✓	$u_{n+1} = au_n + b$ $u_{n+1} = \frac{1}{5}u_n + 12$ $-1 < a < 1$ so a limit exists •[2] ^

Question			Generic scheme	Illustrative scheme	Max mark
3.			•¹ start to differentiate •² complete differentiation	•¹ $7\left(5x^2+3\right)^6\ldots$ •² $\ldots\times 10x$	**2**

Notes:

1. •¹ is awarded for the appearance of $7\left(5x^2+3\right)^6$.
2. For $70x\left(5x^2+3\right)^6$ with no working, award 2/2.
3. Accept $7u^6$ where $u=5x^2+3$ for •¹.
4. Do not award •² where the answer includes '$+c$'.

Commonly Observed Responses:

Candidate A - differentiating over two lines	Candidate B - poor notation
$7\left(5x^2+3\right)^6$ •¹ ✓ $7\left(5x^2+3\right)^6\times 10x$ •² ^	$y=\left(5x^2+3\right)^7$ $y=5x^2+3$ $\frac{dy}{dx}=10x$ $\frac{dy}{dx}=7\left(5x^2+3\right)^6\times 10x$ •¹ ✓ •² ✓
Candidate C - poor communication	**Candidate D - insufficient evidence for •¹**
$y=\left(5x^2+3\right)^7$ $y=7\left(5x^2+3\right)^6\times 10x$ •¹ ✓ •² ✓	$70\left(5x^2+3\right)^6$ •¹ ✗ •² ✗ or $35\left(5x^2+3\right)^6$ •¹ ✗ •² ✗

Question			Generic scheme	Illustrative scheme	Max mark
4.			**Method 1** •¹ interpret ratio •² find coordinates of R	**Method 1** •¹ $\begin{pmatrix}2\\4\\-4\end{pmatrix}, \begin{pmatrix}-2\\-4\\4\end{pmatrix}, \begin{pmatrix}3\\6\\-6\end{pmatrix}$ or $\begin{pmatrix}-3\\-6\\6\end{pmatrix}$ •² $(-4,5,-2)$	2
			Method 2 •¹ interpret ratio •² find coordinates of R	**Method 2** •¹ eg $\overrightarrow{PR}=\frac{2}{5}\overrightarrow{PQ}$, $\overrightarrow{QR}=\frac{3}{5}\overrightarrow{QP}$ or $\overrightarrow{PR}=\frac{2}{3}\overrightarrow{RQ}$ •² $(-4,5,-2)$	
			Method 3 •¹ use section formula •² find coordinates of R	**Method 3** •¹ $\frac{1}{5}(3\mathbf{p}+2\mathbf{q})$ •² $(-4,5,-2)$	

Notes:

1. For $(-4,5,-2)$ without working award 2/2.
2. For $\begin{pmatrix}-4\\5\\-2\end{pmatrix}$ without working award 1/2.
3. For $(-3,7,-4)$ (ratio of 3:2 with working) award 1/2. See Candidate A.
4. For $\begin{pmatrix}-3\\7\\-4\end{pmatrix}$ without working award 0/2.

Commonly Observed Responses:

Candidate A		Candidate B	
$\overrightarrow{PR}=\frac{3}{5}\overrightarrow{PQ}$	•¹ ✗	$\frac{\overrightarrow{PR}}{\overrightarrow{RQ}}=\frac{2}{3}$	•¹ ✓
$R=(-3,7,-4)$	•² ✓₁	$3\overrightarrow{PR}=2\overrightarrow{RQ}$	
		$3(\mathbf{r}-\mathbf{p})=2(\mathbf{q}-\mathbf{r})$	
		$5\mathbf{r}=2\mathbf{q}+3\mathbf{p}$	
		Leading to correct answer of $R=(-4,5,-2)$	•² ✓

Question	Generic scheme	Illustrative scheme	Max mark
4. (continued)			

Candidate C	Candidate D
$\overrightarrow{PQ}=\begin{pmatrix}5\\10\\-10\end{pmatrix}$	$\overrightarrow{PR}=\begin{pmatrix}2\\4\\-4\end{pmatrix}$ $\quad\bullet^1$ ✓
$R=\begin{pmatrix}2\\4\\-4\end{pmatrix}$ $\quad\bullet^1$ ✓	$R(-8,-3,6)$ $\quad\bullet^2$ ✗
$R=\begin{pmatrix}-6\\1\\2\end{pmatrix}+\begin{pmatrix}2\\4\\-4\end{pmatrix}$	
$R=\begin{pmatrix}-4\\5\\-2\end{pmatrix}$	
$R(-4,5,-2)$ $\quad\bullet^2$ ✓	

Candidate E - stepping out using absolute values

2 : 3

5

−6 −1

2 or 3

10

1 11

4 or 6

10

2 −8

4 or 6 $\quad\bullet^1$ ✓

$R(-4,5,-2)$ $\quad\bullet^2$ ✓

Question			Generic scheme	Illustrative scheme	Max mark
5.			**Method 1**	**Method 1**	3
			•¹ equate composite function to x	•¹ $h\left(h^{-1}(x)\right)=x$	
			•² write $h\left(h^{-1}(x)\right)$ in terms of $h^{-1}(x)$	•² $2\left(h^{-1}(x)\right)^3-7=x$	
			•³ state inverse function	•³ $h^{-1}(x)=\sqrt[3]{\frac{x+7}{2}}$	
			Method 2	**Method 2**	
			•¹ write as $y=h(x)$ and start to rearrange	•¹ $y=h(x)\Rightarrow x=h^{-1}(y)$ $y+7=2x^3$	
			•² express x in terms of y	•² $x=\sqrt[3]{\frac{y+7}{2}}$	
			•³ state inverse function	•³ $h^{-1}(y)=\sqrt[3]{\frac{y+7}{2}}$ $\Rightarrow h^{-1}(x)=\sqrt[3]{\frac{x+7}{2}}$	

Notes:

1. In method 1, accept $2\left(h^{-1}(x)\right)^3-7=x$ for •¹ and •².
2. In method 2, accept '$y+7=2x^3$' without reference to $y=h(x)\Rightarrow x=h^{-1}(y)$ at •¹.
3. In method 2, accept $h^{-1}(x)=\sqrt[3]{\frac{x+7}{2}}$ without reference to $h^{-1}(y)$ at •³.
4. In method 2, beware of candidates with working where each line is not mathematically equivalent. See candidates A and B for example.
5. At •³ stage, accept h^{-1} written in terms of any dummy variable.

   For example $h^{-1}(y)=\sqrt[3]{\frac{y+7}{2}}$.
6. $y=\sqrt[3]{\frac{x+7}{2}}$ does not gain •³.
7. $h^{-1}(x)=\sqrt[3]{\frac{x+7}{2}}$ with no working gains 3/3.

Question	Generic scheme	Illustrative scheme	Max mark
5. (continued)			

Commonly Observed Responses:

Candidate A $h(x)=2x^3-7$ $y=2x^3-7$ $x=\sqrt[3]{\frac{y+7}{2}}$ $\bullet^1$ ✓ $\bullet^2$ ✓ $y=\sqrt[3]{\frac{x+7}{2}}$ $\bullet^3$ ✗ $h^{-1}(x)=\sqrt[3]{\frac{x+7}{2}}$	**Candidate B** $h(x)=2x^3-7$ $y=2x^3-7$ $x=2y^3-7$ $\bullet^1$ ✗ $y=\sqrt[3]{\frac{x+7}{2}}$ $\bullet^2$ $\checkmark_1$ $h^{-1}(x)=\sqrt[3]{\frac{x+7}{2}}$ $\bullet^3$ $\checkmark_1$
Candidate C $x=2h(x)^3-7$ $\bullet^1$ ✗ $h(x)=\sqrt[3]{\frac{x+7}{2}}$ $\bullet^2$ $\checkmark_1$ $h^{-1}(x)=\sqrt[3]{\frac{x+7}{2}}$ $\bullet^3$ $\checkmark_1$	**Candidate D - Method 1** $h\left(h^{-1}(x)\right)=2\left(h^{-1}(x)\right)^3-7$ $\bullet^2$ ✓ $x=2\left(h^{-1}(x)\right)^3-7$ $\bullet^1$ ✓ $h^{-1}(x)=\sqrt[3]{\frac{x+7}{2}}$ $\bullet^3$ ✓
Candidate E $x\to x^3\to 2x^3\to 2x^3-7=h(x)$ $\times 2\to -7$ $\therefore +7\to \div 2$ $\bullet^1$ ✓ $\sqrt[3]{\frac{x+7}{2}}$ $\bullet^2$ ✓ $h^{-1}(x)=\sqrt[3]{\frac{x+7}{2}}$ $\bullet^3$ ✓	**Candidate F - BEWARE of incorrect notation** $h'(x)=$ $\bullet^3$ ✗

Question			Generic scheme	Illustrative scheme	Max mark
6.	(a)	(i)	•1 find value of $\cos p$	•1 $\cos p = \frac{2}{\sqrt{5}}$ stated or implied by •2	3
			•2 substitute into the formula for $\sin 2p$	•2 $2 \times \frac{1}{\sqrt{5}} \times \frac{2}{\sqrt{5}}$	
			•3 simplify answer	•3 $\frac{4}{5}$	
		(ii)	•4 evaluate $\cos 2p$	•4 $\frac{3}{5}$	1

Notes:

1. Evidence for •1 may appear in (a)(ii).
2. Where a candidate substitutes an incorrect value for $\cos p$ at •2, •2 may be awarded if the candidate has previously stated this incorrect value or it can be implied by a diagram or Pythagoras calculation. See Candidates A and B.
3. Where a candidate explicitly states a value for $\cos p$, subsequent working must follow from that value for •2 to be awarded.
4. •3 is only available as a consequence of substituting into a valid formula at •2.
5. Do not penalise trigonometric ratios which are less than −1 or greater than 1 throughout this question.

Commonly Observed Responses:

Candidate A – incorrect use of Pythagoras		Candidate B – no evidence of Pythagoras	
$\sqrt{\sqrt{5}^2 + 1^2} = \sqrt{6}$	•1 ✗		•1 ^
$2 \times \frac{1}{\sqrt{5}} \times \frac{\sqrt{6}}{\sqrt{5}}$	•2 ✓1	$2 \times \frac{1}{\sqrt{5}} \times \frac{\sqrt{6}}{\sqrt{5}}$	•2 ✗
$\frac{2\sqrt{6}}{5}$	•3 ✓1	$\frac{2\sqrt{6}}{5}$	•3 ✓1

Candidate C	
$2 \times \sin\frac{1}{\sqrt{5}} \times \cos\frac{2}{\sqrt{5}}$	•1 ✓ •2 ✗
$\frac{4}{5}$	•3 ✗

Question			Generic scheme	Illustrative scheme	Max mark
	(b)		•5 evaluate $\sin 4p$	•5 $\frac{24}{25}$	1

Notes:

6. •5 is only available for an answer expressed as a single fraction.

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
7.			**Method 1**	**Method 1**	4
			•¹ substitute for y	•¹ $x^2+(2x)^2-14x-8(2x)+45=0$	
			•² write in standard quadratic form	•² $5x^2-30x+45=0$	
			•³ determine x-coordinate	•³ 3	
			•⁴ determine y-coordinate	•⁴ 6	
			Method 2	**Method 2**	
			•¹ substitute for x	•¹ $\left(\frac{y}{2}\right)^2+y^2-14\left(\frac{y}{2}\right)-8y+45=0$	
			•² write in standard quadratic form	•² $\frac{5}{4}y^2-15y+45=0$	
			•³ determine y-coordinate	•³ 6	
			•⁴ determine x-coordinate	•⁴ 3	
			Method 3	**Method 3**	
			•¹ use centre and perpendicular gradient to determine equation of radius through point of contact	•¹ $x+2y=15$	
			•² substitute for y	•² $x+2(2x)=15$	
			•³ determine x-coordinate	•³ 3	
			•⁴ determine y-coordinate	•⁴ 6	

Notes:

1. In Methods 1 and 2, treat an absence of brackets at the •¹ stage as bad form only if corrected on the next line of working.
2. In Methods 1 and 2, •¹ is only available if the '=0' appears by the •² stage.
3. In Methods 1 and 2, if a candidate arrives at an equation which is not a quadratic •³ and •⁴ are unavailable.
4. Where the quadratic obtained at •² in Methods 1 and 2, does not have repeated roots •³ and •⁴ are not available.
5. In Method 3 accept $y-4=-\frac{1}{2}(x-7)$, $-\frac{1}{2}=\frac{4-y}{7-x}$ or equivalent for •¹.
6. In Method 3 •², •³ and •⁴ are unavailable to candidates who find the equation of any other line.
7. For (3,6) without working, award 0/4.
8. For answer of (3,6) verified in both equations, or (3,6) generated by the linear equation and verified in the equation of the circle, award 4/4.

Question	Generic scheme	Illustrative scheme	Max mark
7. (continued)			

Commonly Observed Responses:

Candidate A - substitution into the equation of the circle ⋮ $x=3$ $\bullet^3$ ✓ $(3)^2+y^2-14(3)-8y+45=0$ $y^2-8y+12=0$ $(y-2)(y-6)=0$ $y=6$ $\bullet^4$ ✓ no need to explicitly consider $y=2$ **However,** (3,6) and (3,2) $\bullet^4$ ✗	

Question			Generic scheme	Illustrative scheme	Max mark
8.			$\bullet^1$ use discriminant	$\bullet^1$ $(m-4)^2-4(1)(2m-3)$	4
			$\bullet^2$ apply condition	$\bullet^2$ $(m-4)^2-4(1)(2m-3)<0$	
			$\bullet^3$ identify roots of quadratic expression	$\bullet^3$ 2, 14	
			$\bullet^4$ state range with justification	$\bullet^4$ $2<m<14$ with eg labelled sketch or table of signs	

Notes:

1. At $\bullet^1$, treat the inconsistent use of brackets: For example $m-4^2-4(1)(2m-3)$ or $(m-4)^2-4\times1\times2m-3$ as bad form only if the candidate deals with the unbracketed terms correctly in the next line of working.
2. Where candidates express a, b and c in terms of m, and then state $b^2-4ac<0$, award $\bullet^2$.
3. If candidates have the condition 'discriminant >0', 'discriminant ≤ 0' or 'discriminant ≥ 0', then $\bullet^2$ is lost but $\bullet^3$ and $\bullet^4$ are available.
4. Ignore the appearance of $b^2-4ac=0$ where the correct condition has subsequently been applied.
5. If candidates only work with the condition 'discriminant $=0$', then $\bullet^2$ and $\bullet^4$ are unavailable.
6. Accept the appearance of 2 and 14 within inequalities for $\bullet^3$.
7. At $\bullet^4$ accept "$m>2$ and $m<14$" or "$m>2$, $m<14$" together with the required justification.
8. For the appearance of x in any expression of the discriminant, no further marks are available.

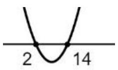
Commonly Observed Responses:

Candidate A - no expressions for *a*, *b* and *c*

No real roots $b^2-4ac<0$

$m^2-16m+28=0$ $\bullet^1$ ✓

$m=2,\ m=14$ $\bullet^3$ ✓

$2<m<14$ 

$\bullet^2$ ✓ $\bullet^4$ ✓

In this case $\bullet^2$ is only available where $\bullet^4$ is awarded

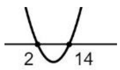
Candidate B

$(m-4)^2-4(1)(2m-3)$ $\bullet^1$ ✓

$m^2-16m+28=0$

$m=2,\ m=14$ $\bullet^3$ ✓

$b^2-4ac<0$

$2<m<14$ 

$\bullet^2$ ✓ $\bullet^4$ ✓

In this case $\bullet^2$ is only available where $\bullet^4$ is awarded

Question	Generic scheme	Illustrative scheme	Max mark

8. (continued)

Candidate C

$(m-4)^2 - 4(1)(2m-3)$ $\bullet^1$ ✓

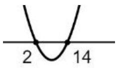
$b^2 - 4ac = 0$

$m^2 - 16m + 28 = 0$

$m = 2,\ m = 14$ $\bullet^3$ ✓

$m^2 - 16m + 28 < 0$ $\bullet^2$ ✓

$2 < m < 14$ $\bullet^4$ ✓



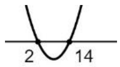
Candidate D

$(m-4)^2 - 4(1)(2m-3)$ $\bullet^1$ ✓

$m^2 - 16m + 28 = 0$ $\bullet^2$ ✗

$m = 2,\ m = 14$ $\bullet^3$ ✓

$2 < m < 14$ $\bullet^4$ $✓_2$



Candidate E - not solving a quadratic

$m - 4^2 - 4(1)(2m-3) < 0$ $\bullet^1$ ✗ $\bullet^2$ ✓ $\bullet^3$ ✗

$-7m - 4 < 0$

$m > -\frac{4}{7}$ $\bullet^4$ $✓_2$

Question			Generic scheme	Illustrative scheme	Max mark
9.			**Method 1** •¹ apply $\log_a x + \log_a y = \log_a xy$ •² apply $m\log_a x = \log_a x^m$ •³ apply $\log_a x - \log_a y = \log_a \frac{x}{y}$ and express in required form	**Method 1** •¹ $\log_a(5\times 80)$… stated or implied by •³ •² $\ldots - \log_a 10^2$ stated or implied by •³ •³ $\log_a 4$	3
			Method 2 •¹ apply $m\log_a x = \log_a x^m$ •² apply $\log_a x - \log_a y = \log_a \frac{x}{y}$ •³ apply $\log_a x + \log_a y = \log_a xy$ and express in required form	**Method 2** •¹ $\ldots - \log_a 10^2$ stated or implied by •³ •² $\ldots + \log_a\left(\frac{80}{10^2}\right)$ stated or implied by •³ •³ $\log_a 4$	

Notes:

1. Where an error at the •¹ or •² stage leads to a non-integer value for k, •³ is still available.
2. Each line of working must be equivalent to the line above within a valid strategy. See commonly observed responses.
3. Where candidates apply the laws of logarithms in the incorrect order see Candidates A and B.
4. Where candidates do not consider the '2', a maximum of 1/3 is available. See Candidate C.
5. Do not penalise the omission of the base of the logarithm.
6. Correct answer with no working, award 3/3.
7. Where candidates form an invalid equation, •¹ and •² may only be awarded for working with $\log_a 5 + \log_a 80 - 2\log_a 10$ on one side of the equation; •³ is not available.

Commonly Observed Responses:

Candidate A

$\log_a 5 + 2\log_a\left(\frac{80}{10}\right)$

$2\log_a\left(\frac{5\times 80}{10}\right)$

$\log_a(40)^2$

$\log_a 1600$

Award 1/3

Candidate B

$\log_a 400 - 2\log_a 10$

$2\log_a\left(\frac{400}{10}\right)$

$\log_a(40)^2$

$\log_a 1600$

Award 2/3

Candidate C - ignoring the 2

$\log_a 5 + \log_a 80 - 2\log_a 10$

$\log_a 5 + \log_a \frac{80}{10}$

$\log_a 40$

Award 1/3

Question			Generic scheme	Illustrative scheme	Max mark
10.	(a)		•¹ use 1 in synthetic division or in evaluation of quartic	•¹ $\begin{array}{r\|rrrrr} 1 & 2 & 3 & -4 & -3 & 2 \\ & & & & & \\ \hline & 2 & & & & \boxed{} \end{array}$ or $2\times(1)^4+3\times(1)^3-4\times(1)^2$ $-3\times(1)+2$	2
			•² complete division/evaluation and interpret result	•² $\begin{array}{r\|rrrrr} 1 & 2 & 3 & -4 & -3 & 2 \\ & & 2 & 5 & 1 & -2 \\ \hline & 2 & 5 & 1 & -2 & \boxed{0} \end{array}$ Remainder $=0\therefore(x-1)$ is a factor or $f(1)=0\therefore(x-1)$ is a factor	

Notes:

1. Communication at •² must be consistent with working at that stage i.e. a candidate's working must arrive legitimately at 0 before •² can be awarded.
2. Accept any of the following for •²:
   - '$f(1)=0$ so $(x-1)$ is a factor'
   - 'since remainder = 0, it is a factor'
   - the '0' from any method linked to the word 'factor' by 'so', 'hence', $\therefore$, $\rightarrow$, $\Rightarrow$ etc.
3. Do not accept any of the following for •²:
   - double underlining the '0' or boxing the '0' without comment
   - '$x=1$ is a factor', '… is a root'
   - the word 'factor' only, with no link.

Commonly Observed Responses:

Candidate A - grid method

	$2x^3$			
x	$2x^4$	$5x^3$		
-1	$-2x^3$			

•¹ ✓

	$2x^3$	$5x^2$	x	-2
x	$2x^4$	$5x^3$	x^2	$-2x$
-1	$-2x^3$	$-5x^2$	$-x$	2

'with no remainder'

$\therefore(x-1)$ is a factor •² ✓

Candidate B - grid method

	$2x^3$			
x	$2x^4$	$5x^3$		
-1	$-2x^3$			

•¹ ✓

	$2x^3$	$5x^2$	x	-2
x	$2x^4$	$5x^3$	x^2	$-2x$
-1	$-2x^3$	$-5x^2$	$-x$	2

$\therefore(x-1)(2x^3+5x^2+x-2)=2x^4+3x^3-4x^2-3x+2$

$\therefore(x-1)$ is a factor •² ✓

<table>
<tr><th colspan="3">Question</th><th>Generic scheme</th><th>Illustrative scheme</th><th>Max mark</th></tr>
<tr><td rowspan="4">10.</td><td rowspan="4">(b)</td><td rowspan="4"></td><td>$\bullet^3$ identify cubic and attempt to factorise</td><td>$\bullet^3$ eg
$\begin{array}{r|rrrr} -1 & 2 & 5 & 1 & -2 \\ & & -2 & -3 & \\ \hline & 2 & 3 & & \boxed{} \end{array}$
or
$\begin{array}{r|rrrr} -2 & 2 & 5 & 1 & -2 \\ & & -4 & -2 & \\ \hline & 2 & 1 & & \boxed{} \end{array}$</td><td rowspan="4">4</td></tr>
<tr><td>$\bullet^4$ find second factor</td><td>$\bullet^4$ eg
$\begin{array}{r|rrrr} -1 & 2 & 5 & 1 & -2 \\ & & -2 & -3 & 2 \\ \hline & 2 & 3 & -2 & \boxed{0} \end{array}$
leading to $(x+1)$
or
$\begin{array}{r|rrrr} -2 & 2 & 5 & 1 & -2 \\ & & -4 & -2 & 2 \\ \hline & 2 & 1 & -1 & \boxed{0} \end{array}$
leading to $(x+2)$</td></tr>
<tr><td>$\bullet^5$ identify quadratic</td><td>$\bullet^5$ $2x^2+3x-2$ or $2x^2+x-1$</td></tr>
<tr><td>$\bullet^6$ complete factorisation</td><td>$\bullet^6$ $(x-1)(x+1)(2x-1)(x+2)$
stated explicitly</td></tr>
<tr><td colspan="6">Notes:</td></tr>
<tr><td colspan="6">4. Ignore the appearance of ‘$=0$’.
5. Candidates who arrive at $(x-1)(x+1)(2x^2+3x-2)$ or $(x-1)(x+2)(2x^2+x-1)$ by using algebraic long division or by inspection, gain $\bullet^3$, $\bullet^4$ and $\bullet^5$.
6. Where a candidate only identifies additional factors from a quartic, only $\bullet^4$ is available.
7. $\bullet^3$ and $\bullet^4$ may be awarded for applications of synthetic division even when previous trials contain errors. $\bullet^5$ and $\bullet^6$ are available.</td></tr>
</table>

Question	Generic scheme	Illustrative scheme	Max mark
10. (b) (continued)			

Commonly Observed Responses:

Candidate C - grid method

(a)

	$2x^3$	$5x^2$	x	-2
x	$2x^4$	$5x^3$	x^2	$-2x$
-1	$-2x^3$	$-5x^2$	$-x$	2

(b)

	$2x^2$	…	…
x	$2x^3$	…	…
…	…	…	…

$\bullet^3$ ✓

$\bullet^3$ is awarded for evidence of the cubic expression (which may be in the grid from part (a)) **AND** the terms in the diagonal boxes summing to the second and third terms in the cubic respectively.

	$2x^2$	$3x$	-2
x	$2x^3$	$3x^2$	$-2x$
$+1$	$2x^2$	$3x$	-2

$\bullet^4$ ✓

$2x^2+3x-2$ $\bullet^5$ ✓

$(x-1)(x+1)(2x-1)(x+2)$ $\bullet^6$ ✓

Candidate D - grid method

(a)

	$2x^3$	$5x^2$	x	-2
x	$2x^4$	$5x^3$	x^2	$-2x$
-1	$-2x^3$	$-5x^2$	$-x$	2

(b)

	$2x^2$	…	…
x	$2x^3$	…	…
…	…	…	…

$\bullet^3$ ✓

$\bullet^3$ is awarded for evidence of the cubic expression (which may be in the grid from part (a)) **AND** the terms in the diagonal boxes summing to the second and third terms in the cubic respectively.

	$2x^2$	x	-1
x	$2x^3$	x^2	$-x$
$+2$	$4x^2$	$2x$	-2

$\bullet^4$ ✓

$2x^2+x-1$ $\bullet^5$ ✓

$(x-1)(x+2)(x+1)(2x-1)$ $\bullet^6$ ✓

Candidate E

$$\begin{array}{c|cccc} \frac{1}{2} & 2 & 5 & 1 & -2 \\ & & 1 & 3 & 2 \\ \hline & 2 & 6 & 4 & \boxed{0} \end{array}$$

$\bullet^3$ ✓ $\bullet^4$ ✓

$\left(x-\frac{1}{2}\right)\left(2x^2+6x+4\right)$ $\bullet^5$ ✓

$(2x-1)\left(x^2+3x+2\right)$

$(x-1)(2x-1)(x+1)(x+2)$ $\bullet^6$ ✓

Candidate F

$$\begin{array}{c|cccc} \frac{1}{2} & 2 & 5 & 1 & -2 \\ & & 1 & 3 & 2 \\ \hline & 2 & 6 & 4 & \boxed{0} \end{array}$$

$\bullet^3$ ✓ $\bullet^4$ ✓

$\left(x-\frac{1}{2}\right)\left(2x^2+6x+4\right)$ $\bullet^5$ ✓

$\left(x-\frac{1}{2}\right)(2x+2)(x+2)$

$(x-1)\left(x-\frac{1}{2}\right)(2x+2)(x+2)$ $\bullet^6$ ^

Question			Generic scheme	Illustrative scheme	Max mark
11.	(a)		•¹ use compound angle formula	•¹ $k\cos x^\circ \cos a^\circ + k\sin x^\circ \sin a^\circ$ **stated explicitly**	4
			•² compare coefficients	•² $k\cos a^\circ = 1, k\sin a^\circ = \sqrt{3}$ **stated explicitly**	
			•³ process for k	•³ $k = 2$	
			•⁴ process for a and express in required form	•⁴ $2\cos(x-60)^\circ$	

Notes:

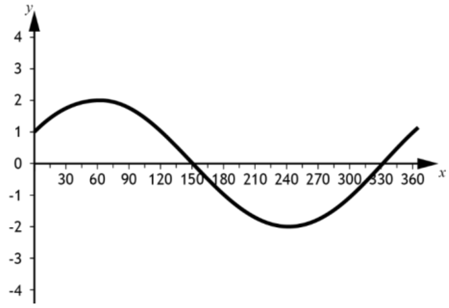
1. Accept $k(\cos x^\circ \cos a^\circ + \sin x^\circ \sin a^\circ)$ for •¹. Treat $k\cos x^\circ \cos a^\circ + \sin x^\circ \sin a^\circ$ as bad form only if the equations at the •² stage both contain k.
2. Do not penalise the omission of degree signs.
3. $2\cos x^\circ \cos a^\circ + 2\sin x^\circ \sin a^\circ$ or $2(\cos x^\circ \cos a^\circ + \sin x^\circ \sin a^\circ)$ is acceptable for •¹ and •³.
4. •² is not available for $k\cos x^\circ = 1, k\sin x^\circ = \sqrt{3}$, however •⁴ may still be gained- see Candidate E
5. •³ is only available for a single value of $k, k > 0$.
6. •³ is not available to candidates who work with $\sqrt{4}$ throughout parts (a) and (b) without explicitly simplifying at any stage. •⁴ is still available.
7. •⁴ is not available for a value of a given in radians.
8. Candidates may use any form of the wave function for •¹, •² and •³. However, •⁴ is only available if the wave is interpreted in the form $k\cos(x-a)^\circ$.
9. Evidence for •⁴ may not appear until part (b).

Commonly Observed Responses:

Candidate A	Candidate B - inconsistency	Candidate C
•¹ ^	$k\cos x^\circ \cos a^\circ + k\sin x^\circ \sin a^\circ$ •¹ ✓	$\cos x^\circ \cos a^\circ + \sin x^\circ \sin a^\circ$ •¹ ✗
$2\cos a^\circ = 1$	$\cos a^\circ = 1$	$\cos a^\circ = 1$
$2\sin a^\circ = \sqrt{3}$ •² ✓ •³ ✓	$\sin a^\circ = \sqrt{3}$ •² ✗	$\sin a^\circ = \sqrt{3}$ •² ✓2
		$k = 2$ •³ ✓
$\tan a^\circ = \sqrt{3}$	$\tan a^\circ = \sqrt{3}$	$\tan a^\circ = \sqrt{3}$
$a = 60$	$a = 60$	$a = 60$
$2\cos(x-60)^\circ$ •⁴ ✓	$2\cos(x-60)^\circ$ •³ ✓ •⁴ ✗	$2\cos(x-60)^\circ$ •⁴ ✗

Question	Generic scheme	Illustrative scheme	Max mark
11. (a) (continued)			

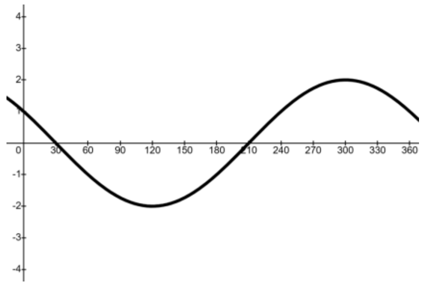
Candidate D - errors at •2	Candidate E - use of x at •2	Candidate F
$k\cos x°\cos a° + k\sin x°\sin a°$ •1 ✓ $k\cos a° = \sqrt{3}$ $k\sin a° = 1$ •2 ✗ $\tan a° = \frac{1}{\sqrt{3}}$ $a = 30$ $2\cos(x-30)°$ •3 ✓ •4 ✓1	$k\cos x°\cos a° + k\sin x°\sin a°$ •1 ✓ $k\cos x° = 1$ $k\sin x° = \sqrt{3}$ •2 ✗ $\tan x° = \sqrt{3}$ $x = 60$ $2\cos(x-60)°$ •3 ✓ •4 ✓1	$k\sin A\cos B + k\cos A\sin B$ •1 ✗ $k\cos A = 1$ $k\sin A = \sqrt{3}$ •2 ✗ $\tan A = \sqrt{3}$ $2\cos(x-60)°$ •3 ✓ •4 ✓1

Question			Generic scheme	Illustrative scheme	Max mark
11.	(b)		•5 exactly two roots identifiable from graph •6 coordinates of exactly two turning points identifiable from graph •7 y-intercept and value of y at $x = 360$ identifiable from graph	•5 $(150,0)$ and $(330,0)$ •6 $(60,2)$ and $(240,-2)$ •7 1 	**3**

Notes:

10. •5, •6 and •7 are only available for attempting to draw a "cosine" graph with a period of $360°$.
11. Ignore any part of a graph drawn outwith $0 \le x \le 360$.
12. Vertical marking is not applicable to •5 and •6.
13. Candidate's sketch in (b) must be consistent with the equation obtained in (a), see also Candidates G and H.
14. For any incorrect horizontal translation of the graph of the wave function arrived at in part (a) only •6 is available.

Commonly Observed Responses:

Candidate G - incorrect translation	Candidate H - incorrect equation
(a) $2\cos(x-60)°$ - correct equation (b) Incorrect translation: Sketch of $2\cos(x+60)°$ only •6 is available	(a) $2\cos(x+60)°$ - incorrect equation (b) Sketch of $2\cos(x+60)°$ all 3 marks available 

Question			Generic scheme	Illustrative scheme	Max mark
12.			•¹ write in differentiable form •² differentiate •³ solve for $a^{-\frac{2}{3}}$ or $a^{\frac{2}{3}}$ •⁴ solve for a	•¹ $12x^{\frac{1}{3}}$ stated or implied by •² •² $12\times\frac{1}{3}\times x^{-\frac{2}{3}}$ •³ $a^{-\frac{2}{3}}=\frac{1}{4}$ or $a^{\frac{2}{3}}=4$ •⁴ $a=8$	4

Notes:

1. •² is only available for differentiating a term with a fractional index.
2. Where candidates attempt to integrate or make no attempt to differentiate, only •¹ is available.
3. Accept $x^{-\frac{2}{3}}=\frac{1}{4}$ or $x^{\frac{2}{3}}=4$ at •³. See Candidates A and B.
4. •⁴ is only available where the expression at •² is of the form $kx^{-\frac{m}{n}}$ where $m\neq 1$.
5. Do not penalise the inclusion of -8 at •⁴.

Commonly Observed Responses:

Candidate A - working in terms of x throughout … •¹ ✓ •² ✓ $x^{-\frac{2}{3}}=\frac{1}{4}$ •³ ✓ $x=8$ •⁴ ✗	**Candidate B** … •¹ ✓ •² ✓ $x^{-\frac{2}{3}}=\frac{1}{4}$ •³ ✓ $(x=8)$ $a=8$ •⁴ ✓
Candidate C $f(x)=12x^{\frac{3}{2}}$ •¹ ✗ $f'(x)=18x^{\frac{1}{2}}$ •² ✓₁ $a^{\frac{1}{2}}=\frac{1}{18}$ •³ ✓₁ $a=\frac{1}{324}$ •⁴ ✓₂	**Candidate D - partly differentiated** $f(x)=12x^{\frac{1}{3}}$ •¹ ✓ $f'(x)=12\times\frac{1}{3}x^{\frac{4}{3}}$ •² ✗ $1=4a^{\frac{4}{3}}$ $\frac{1}{4}=a^{\frac{4}{3}}$ •³ ✓₁ $a=\frac{1}{\sqrt{8}}$ •⁴ ✓₂

Question			Generic scheme	Illustrative scheme	Max mark
13.	(a)		•1 find midpoint of PQ	•1 (5,6)	4
			•2 find gradient of PQ	•2 -4 or $-\frac{8}{2}$	
			•3 find perpendicular gradient	•3 $\frac{1}{4}$	
			•4 find equation of perpendicular bisector	•4 $4y = x + 19$	

Notes:

1. •4 is only available as a consequence of using a perpendicular gradient and a mid-point.
2. The gradient of the perpendicular bisector must appear in fully simplified form at •3 or •4 stage for •3 to be awarded.
3. At •4 accept $4y - x = 19$, $4y - x - 19 = 0$, or any other rearrangement of the equation where the constant terms have been simplified.

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
	(b)		•5 identify x-coordinate of centre	•5 9	4
			•6 find y-coordinate of centre	•6 7	
			•7 find radius	•7 $\sqrt{34}$	
			•8 state equation of circle	•8 $(x-9)^2 + (y-7)^2 = 34$	

Notes:

4. Do not accept "centre = (9,2)" as evidence of •5.
5. Where candidates use PQ, QR or PR as the diameter of the circle no marks are available.
6. •7 and •8 are only available as a consequence of using the point of intersection of two perpendicular bisectors and a point on the circumference of the circle.
7. Accept $r^2 = 34$ for •7.
8. $(x-9)^2 + (y-7)^2 = \left(\sqrt{34}\right)^2$ does not gain •8.

Commonly Observed Responses:

Candidate A - horizontal line through midpoint of PQ	Candidate B - perpendicular bisector of PR
Centre = (9,6) •5 ✓ •6 ✗	Perpendicular bisector of PR: $y = x - 2$
Radius = 5 •7 ✗	Centre = (9,7) •5 ✓ •6 ✓
Equation: $(x-9)^2 + (y-6)^2 = 25$ •8 ✗	⋮

[END OF MARKING INSTRUCTIONS]

H

National
Qualifications
2024

2024 Mathematics

Higher - Paper 2

Question Paper Finalised Marking Instructions

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Question			Generic scheme	Illustrative scheme	Max mark
1.	(a)		$\bullet^1$ determine midpoint of AC	$\bullet^1$ (4,4)	3
			$\bullet^2$ determine gradient of median	$\bullet^2$ 2 or $\frac{10}{5}$	
			$\bullet^3$ find equation of median	$\bullet^3$ $y=2x-4$	

Notes:

1. $\bullet^2$ is only available to candidates who use a midpoint to find a gradient.
2. $\bullet^3$ is only available as a consequence of using a 'midpoint' of AC and the point B
3. At $\bullet^3$ accept any arrangement of a candidate's equation where the constant terms have been simplified.
4. $\bullet^3$ is not available as a consequence of using a perpendicular gradient.

Commonly Observed Responses:

Candidate A - perpendicular bisector of AC

Midpoint = (4,4) $\bullet^1$ ✓

$m_{AC}=-\frac{4}{7}\Rightarrow m_{\perp}=\frac{7}{4}$ $\bullet^2$ ×

$4y=7x-12$ $\bullet^3$ $\checkmark_2$

For other perpendicular bisectors award 0/3

Candidate B - altitude through B

$m_{AC}=-\frac{4}{7}$ $\bullet^1$ ^

$m_{\perp}=\frac{7}{4}$ $\bullet^2$ ×

$4y=7x-17$ $\bullet^3$ $\checkmark_2$

Candidate C - median through A

midpoint BC = $(5,-3)$ $\bullet^1$ ×

$m_{AM}=-\frac{11}{8}$ $\bullet^2$ $\checkmark_1$

$8y=-11x+31$ $\bullet^3$ $\checkmark_2$

Candidate D - median through C

midpoint AB $(-2,1)$ $\bullet^1$ ×

$m_{CM}=-\frac{1}{13}$ $\bullet^2$ $\checkmark_1$

$13y=-x+11$ $\bullet^3$ $\checkmark_2$

Question			Generic scheme	Illustrative scheme	Max mark
	(b)		$\bullet^4$ determine gradient of BC	$\bullet^4$ $\frac{6}{12}$	3
			$\bullet^5$ determine gradient of L	$\bullet^5$ $-\frac{12}{6}$	
			$\bullet^6$ find equation of L	$\bullet^6$ $y=-2x+22$	

Notes:

5. $\bullet^6$ is only available as a consequence of using a perpendicular gradient and C.
6. At $\bullet^6$ accept any arrangement of a candidate's equation where the constant terms have been simplified.

Commonly Observed Responses:

Candidate E - altitude through C

$m_{AB}=-7$ $\bullet^4$ ×

$m_{\perp}=\frac{1}{7}$ $\bullet^5$ $\checkmark_1$

$y=\frac{1}{7}(x-11)$ $\bullet^6$ $\checkmark_1$

Question			Generic scheme	Illustrative scheme	Max mark
1.	(c)		$\bullet^7$ determine x-coordinate $\bullet^8$ determine y-coordinate	$\bullet^7$ 6.5 or $\frac{13}{2}$ $\bullet^8$ 9	**2**

Notes:

7. For $\left(\frac{26}{4}, 9\right)$ award 1/2.

Commonly Observed Responses:

Candidate F - rounding decimals

(a) $4y = 5x - 19$

(b) $y = -2x + 22$

(c) $x = \frac{107}{13} = 8.2$ $\bullet^7$ ✓$_1$

$y = 5.6$ $\bullet^8$ ✓$_1$

Question			Generic scheme	Illustrative scheme	Max mark
2.			•¹ find y-coordinate •² write in differentiable form •³ differentiate •⁴ find gradient of tangent •⁵ determine equation of tangent	•¹ 1 •² $8x^{-3}$ •³ $8\times(-3)x^{-4}$ •⁴ $-\frac{3}{2}$ •⁵ $3x+2y=8$	**5**

Notes:

1. Only •¹ and •² are available to candidates who integrate. However, see Candidates E and F.
2. $8\times(-3)x^{-4}$ without previous working gains •² and •³.
3. •³ is only available for differentiating a negative power. •⁴ and •⁵ are still available.
4. •⁴ is not available for $y=-\frac{3}{2}$. However, where $-\frac{3}{2}$ is then used as the gradient of the straight line, •⁴ may be awarded - see Candidates A, B and C.
5. •⁵ is only available where candidates attempt to find the gradient by substituting into their derivative.
6. •⁵ is not available as a consequence of using a perpendicular gradient.
7. Where $x=2$ has not been used to determine the y-coordinate, •⁵ is not available.

Commonly Observed Responses:

Candidate A - incorrect notation

$y=1$	•¹ ✓ - BoD
$y=8x^{-3}$	•² ✓
$y=-24x^{-4}$	•³ ✓
$y=-\frac{3}{2}$	•⁴ ✓ - BoD
$3x+2y=8$	•⁵ ✓

Candidate B - use of values in equation

$y=1$	•¹ ✓ - BoD
$y=8x^{-3}$	•² ✓
$\frac{dy}{dx}=8\times(-3)x^{-4}$	•³ ✓
$\frac{dy}{dx}=-\frac{3}{2}$	•⁴ ✓
$y=-\frac{3}{2}$	
$3x+2y=8$	•⁵ ✓

Candidate C - incorrect notation

$y=1$	•¹ ✓ - BoD
$y=8x^{-3}$	•² ✓
$\frac{dy}{dx}=8\times(-3)x^{-4}$	•³ ✓
$y=-\frac{3}{2}$	•⁴ ✗

Evidence for •⁴ would need to appear in the equation of the line

Candidate D

$y=1$	•¹ ✓
$y=8x^{-3}$	•² ✓
$\frac{dy}{dx}=8\times(-3)x^{-4}=0$	•³ ✓
$8\times(-3)(2)^{-4}=0$	
$m=-\frac{3}{2}$	•⁴ ✗
$3x+2y=8$	•⁵ ✓₁

Question	Generic scheme	Illustrative scheme	Max mark
2. (continued)			

Candidate E - integrating in part		Candidate F - appearance of $+c$	
$y=1$	$\bullet^1$ ✓	$y=1$	$\bullet^1$ ✓
$y=8x^{-3}$	$\bullet^2$ ✓	$y=8x^{-3}$	$\bullet^2$ ✓
$\frac{dy}{dx}=-24x^{-2}$	$\bullet^3$ ✗	$\frac{dy}{dx}=-24x^{-4}+c$	$\bullet^3$ ✗ $\bullet^4$ ✗
$\frac{dy}{dx}=-6$	$\bullet^4$ ✓$_1$		$\bullet^5$ ✗
$y=-6x+13$	$\bullet^5$ ✓$_1$		

Question			Generic scheme	Illustrative scheme	Max mark
3.	(a)		•[1] find $\overrightarrow{ED}$	•[1] $\begin{pmatrix} 1 \\ -4 \\ 6 \end{pmatrix}$	**2**
			•[2] find $\overrightarrow{EF}$	•[2] $\begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix}$	
Notes:					
1. For candidates who find **both** $\overrightarrow{DE}$ **and** $\overrightarrow{FE}$ correctly, award 1/2. 2. Accept vectors written horizontally.					
Commonly Observed Responses:					
	(b)	(i)	•[3] evaluate $\overrightarrow{ED}.\overrightarrow{EF}$	•[3] 16	**1**
		(ii)	•[4] evaluate $\lvert\overrightarrow{ED}\rvert$	•[4] $\sqrt{53}$	**4**
			•[5] evaluate $\lvert\overrightarrow{EF}\rvert$	•[5] $\sqrt{14}$	
			•[6] substitute into formula for scalar product	•[6] $\cos \text{DEF} = \dfrac{16}{\sqrt{53}\times\sqrt{14}}$ or $\sqrt{53}\times\sqrt{14}\times\cos \text{DEF} = 16$	
			•[7] calculate angle	•[7] 54.028…° or 0.942… radians	

Question	Generic scheme	Illustrative scheme	Max mark

3. (b) (continued)

Notes:

3. Do not penalise candidates who treat negative signs with a lack of rigour when calculating a magnitude. For example accept $\sqrt{1^2+4^2+6^2}=\sqrt{53}$ or $\sqrt{1^2+-4^2+6^2}=\sqrt{53}$ for $\bullet^4$.
   However, do not accept $\sqrt{1^2-4^2+6^2}=\sqrt{53}$ for $\bullet^4$.
4. $\bullet^6$ is not available to candidates who simply state the formula $\cos\theta=\dfrac{\overrightarrow{ED}.\overrightarrow{EF}}{|\overrightarrow{ED}||\overrightarrow{EF}|}$.

   However, $\cos\theta=\dfrac{16}{\sqrt{53}\times\sqrt{14}}$ and $\sqrt{53}\times\sqrt{14}\times\cos\theta=16$ are acceptable for $\bullet^6$.
5. Accept correct answers rounded to 54° or 0.9 radians (or 60 gradians).
6. Do not penalise the omission or incorrect use of units.
7. $\bullet^7$ is only available as a result of using a valid strategy.
8. $\bullet^7$ is only available for a single angle.
9. For a correct answer with no working award 0/4.

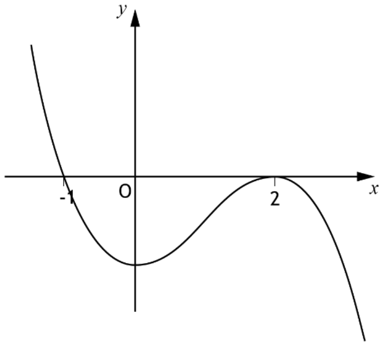
Commonly Observed Responses:

Candidate A - poor notation		Candidate B - insufficient communication	
$\begin{pmatrix}1\\-4\\6\end{pmatrix}\cdot\begin{pmatrix}2\\1\\3\end{pmatrix}=\begin{pmatrix}2\\-4\\18\end{pmatrix}=16$	$\bullet^3$ ✗	$\lvert\overrightarrow{ED}\rvert=\sqrt{53}$	$\bullet^4$ ✓
		$\lvert\overrightarrow{EF}\rvert=\sqrt{14}$	$\bullet^5$ ✓
		$\dfrac{16}{\sqrt{53}\times\sqrt{14}}$	$\bullet^6$ ^
		54.028…° or 0.942… radians	$\bullet^7$ ✓$_1$
Candidate C - BEWARE			
$\lvert\overrightarrow{OF}\rvert=\sqrt{14}$	$\bullet^5$ ✗		

Question			Generic scheme	Illustrative scheme	Max mark
4.	(a)		•[1] identify x-coordinate •[2] identify y-coordinate	•[1] 3 •[2] 5	2

Notes:

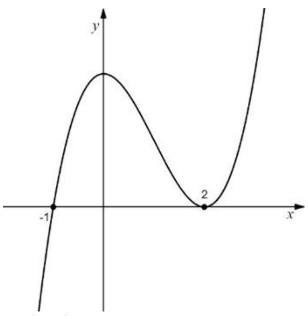
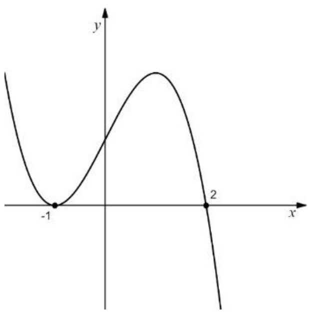
Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
	(b)		•[3] identify roots •[4] interpret point of inflection •[5] identify orientation and complete cubic curve	•[3] "cubic" with roots at −1 and 2 •[4] "cubic" with turning point at (2,0) •[5] cubic with maximum turning point at (2,0) 	3

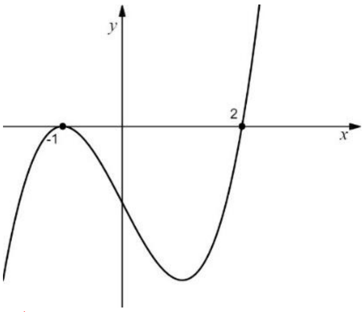
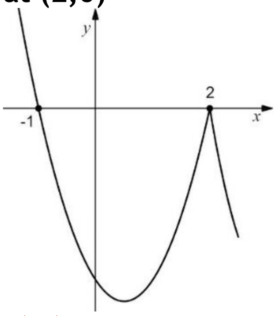
Notes:

1. Note that the position of the minimum turning point of $f'(x)$ is not being assessed.
2. Where a candidate has not drawn a cubic curve or their graph does not extend outwith $-1 \le x \le 2$ award 0/3. However see Candidate D.
3. Do not penalise the appearance of an additional root outwith $-1 \le x \le 2$ (on a cubic curve) at •[3].

Commonly Observed Responses:

Candidate A - $-f'(x)$	Candidate B
 ✓ ✓ ✗	 ✓ ✗ ✗

Question	Generic scheme	Illustrative scheme	Max mark
4. (b) (continued)			

Candidate C  ✓ × ×	**Candidate D - left derivative ≠ right derivative at (2,0)**  ✓ ✓ ×

Question			Generic scheme	Illustrative scheme	Max mark
5.			$\bullet^1$ integrate $\bullet^2$ substitute limits $\bullet^3$ evaluate integral	$\bullet^1$ $-\frac{1}{5}\cos 5x$ $\bullet^2$ $\left(-\frac{1}{5}\cos\left(5\times\frac{\pi}{7}\right)\right)-\left(-\frac{1}{5}\cos(5\times 0)\right)$ $\bullet^3$ 0.3246...	3

Notes:

1. For candidates who differentiate throughout, make no attempt to integrate, or use another invalid approach (for example $\cos 5x^2$) award 0/3.
2. Do not penalise the inclusion of '$+c$' or the continued appearance of the integral sign after integrating.
3. Accept $\left(-\frac{1}{5}\cos 5\left(\frac{\pi}{7}\right)\right)-\left(-\frac{1}{5}\cos 5(0)\right)$ for $\bullet^2$.
4. $\bullet^3$ is only available where candidates have considered both limits within a trigonometric function.

Commonly Observed Responses:

Candidate A - integrated in part $-\cos 5x$ $\bullet^1$ ✗ $-\cos\left(\frac{5\pi}{7}\right)-\left(-\cos(5\times 0)\right)$ $\bullet^2$ $\checkmark_1$ 1.623... $\bullet^3$ $\checkmark_1$	**Candidate B - insufficient evidence of integration** $\cos 5x$ $\bullet^1$ ✗ $\cos\left(\frac{5\pi}{7}\right)-\left(\cos(5\times 0)\right)$ $\bullet^2$ $\checkmark_2$ −1.623 $\bullet^3$ $\checkmark_2$
Candidate C - insufficient evidence of integration $\frac{1}{5}\sin 5x$ $\bullet^1$ ✗ $\frac{1}{5}\sin\frac{5\pi}{7}-\frac{1}{5}\sin 0$ $\bullet^2$ $\checkmark_2$ 0.156... $\bullet^3$ $\checkmark_2$	**Candidate D - working in degrees before integrating** $\int_0^{25.7...}\sin 5x\,dx$ $\bullet^1$ ✗ $-\frac{1}{5}\cos 5x$ $\left(-\frac{1}{5}\cos 128.57...\right)-\left(-\frac{1}{5}\cos 0\right)$ $\bullet^2$ $\checkmark_1$ 0.3246... $\bullet^3$ $\checkmark_1$

Question			Generic scheme	Illustrative scheme	Max mark
6.			**Method 1**	**Method 1**	5
			•1 state linear equation	•1 $\log_5 y = 3\log_5 x - 2$	
			•2 introduce logs	•2 $\log_5 y = 3\log_5 x - 2\log_5 5$	
			•3 use laws of logs	•3 $\log_5 y = \log_5 x^3 - \log_5 5^2$	
			•4 use laws of logs	•4 $\log_5 y = \log_5 \frac{x^3}{5^2}$	
			•5 state a and b	•5 $a = \frac{1}{25}, b = 3$ or $y = \frac{x^3}{25}$	
			Method 2	**Method 2**	5
			•1 state linear equation	•1 $\log_5 y = 3\log_5 x - 2$	
			•2 use laws of logs	•2 $\log_5 y = \log_5 x^3 - 2$	
			•3 use laws of logs	•3 $\log_5 \frac{y}{x^3} = -2$	
			•4 use laws of logs	•4 $\frac{y}{x^3} = 5^{-2}$	
			•5 state a and b	•5 $a = \frac{1}{25}, b = 3$ or $y = \frac{x^3}{25}$	
			Method 3	**Method 3** **The equations at •1, •2 and •3 must be stated explicitly**	5
			•1 introduce logs to $y = ax^b$	•1 $\log_5 y = \log_5 ax^b$	
			•2 use laws of logs	•2 $\log_5 y = b\log_5 x + \log_5 a$	
			•3 interpret intercept	•3 $\log_5 a = -2$	
			•4 use laws of logs	•4 $a = \frac{1}{25}$	
			•5 interpret gradient	•5 $b = 3$	

Question	Generic scheme	Illustrative scheme	Max mark
6. (continued)			

Notes

1. In any method, marks may only be awarded within a valid strategy using $y = ax^b$. For example, see Candidates C and D.
2. Markers must identify the method which best matches the candidate's approach; markers must not mix and match between methods.
3. Penalise the omission of base 5 at most once in any method.
4. Where candidates use an incorrect base then only $\bullet^2$ **and** $\bullet^3$ are available (in any method).
5. Do not accept $a = 5^{-2}$.
6. In Method 3, do not accept $m = 3$ or gradient $= 3$ for $\bullet^5$.
7. Do not penalise candidates who score out "log" from equations of the form $\log_5 m = \log_5 n$.

Commonly Observed Responses

Candidate A - missing equations at $\bullet^1$, $\bullet^2$ and $\bullet^3$ in Method 3 $a = \frac{1}{25}$ $\bullet^4$ ✓ $b = 3$ $\bullet^5$ ✓	**Candidate B - no working - Method 3** $b = \frac{1}{25}$ $\bullet^4$ ✗ $a = 3$ $\bullet^5$ ✗
Candidate C - Method 2 $y = 3x - 2$ $\log_5 y = 3\log_5 x - 2$ $\bullet^1$ ✓ $\log_5 y = \log_5 x^3 - 2$ $\bullet^2$ ✓ $y = x^3 - 2$ $\bullet^3$ ✗ $\bullet^4$ ✗ $\bullet^5$ ✗	**Candidate D - Method 2** $\log_5 y = 3\log_5 x - 2$ $\bullet^1$ ✓ $\log_5 y = \log_5 x^3 - 2$ $\bullet^2$ ✓ $\frac{y}{x^3} = -2$ $\bullet^3$ ✗ $\bullet^4$ ✗ $\bullet^5$ ✗
Candidate E - use of coordinate pairs $\log_5 x = 4$ **and** $\log_5 y = 10$ $\bullet^1$ ✓ $x = 5^4$ and $y = 5^{10}$ $\bullet^2$ ✓ $\log_5 x = 0$, $\log_5 y = -2$ $\Rightarrow x = 1$, $y = 5^{-2}$ $\bullet^3$ ✓ $5^{-2} = a \times 1^b \Rightarrow a = \frac{1}{25}$ $\bullet^4$ ✓ $5^{10} = 5^{-2} \times 5^{4b} \Rightarrow -2 + 4b = 10$ $\Rightarrow b = 3$ $\bullet^5$ ✓ Candidates may use $(0,-2)$ for $\bullet^1$ and $\bullet^2$ and $(4,10)$ for $\bullet^3$.	

Question			Generic scheme	Illustrative scheme	Max mark
7.			**Method 1**	**Method 1**	5
			•1 integrate using 'upper' – 'lower'	•1 $\int\left(\left(6+4x-2x^2\right)-\left(x^3-6x^2+11x\right)\right)dx$	
			•2 identify limits	•2 $\int_0^2\left(\left(6+4x-2x^2\right)-\left(x^3-6x^2+11x\right)\right)dx$	
			•3 integrate	•3 $6x-\frac{7}{2}x^2+\frac{4}{3}x^3-\frac{1}{4}x^4$	
			•4 substitute limits	•4 $\left(6(2)-\frac{7}{2}(2)^2+\frac{4}{3}(2)^3-\frac{1}{4}(2)^4\right)-0$	
			•5 evaluate area	•5 $\frac{14}{3}$ (units^2)	
			Method 2	**Method 2**	
			•1 know to integrate between appropriate limits for both equations	•1 $\int_0^2 \ldots dx$ and $\int_0^2 \ldots dx$	
			•2 integrate both functions	•2 $6x+\frac{4x^2}{2}-\frac{2x^3}{3}$ and $\frac{x^4}{4}-\frac{6x^3}{3}+\frac{11x^2}{2}$	
			•3 substitute limits into both expressions	•3 $\left(6(2)+\frac{4(2)^2}{2}-\frac{2(2)^3}{3}\right)-0$ and $\left(\frac{(2)^4}{4}-\frac{6(2)^3}{3}+\frac{11(2)^2}{2}\right)-0$	
			•4 evaluate both integrals	•4 $\frac{44}{3}$ and 10	
			•5 evidence of subtracting areas	•5 $\frac{14}{3}$ (units^2)	

Question	Generic scheme	Illustrative scheme	Max mark

7. (continued)

Notes:

1. Correct answer with no working - award 1/5.
2. Do not penalise lack of 'dx' at $\bullet^1$ in Method 1.
3. In Method 1, limits and 'dx' must appear by the $\bullet^2$ stage for $\bullet^2$ to be awarded and in Method 2 by the $\bullet^1$ stage for $\bullet^1$ to be awarded.
4. In Method 1, treat the absence of brackets at $\bullet^1$ stage as bad form only if the correct integrand is obtained. See Candidates C and D.
5. Where a candidate differentiates one or more terms, or fails to integrate, no further marks are available.
6. In Method 1, accept unsimplified expressions such as $6x+\frac{4x^2}{2}-\frac{2x^3}{3}-\frac{x^4}{4}+\frac{6x^3}{3}-\frac{11x^2}{2}$ at $\bullet^3$.
7. Do not penalise the inclusion of '$+c$'.
8. Do not penalise the continued appearance of the integral sign or dx after integrating.
9. $\bullet^5$ is not available where solutions include statements such as '$-\frac{14}{3}=\frac{14}{3}$ square units'. See Candidates A and B.
10. In Method 1, where a candidate uses an invalid strategy the only marks available are $\bullet^3$ for integrating a polynomial with at least four terms (of different degree) and $\bullet^4$ for substituting the limits of 0 and 2 into the resulting expression. However, see Candidate E.
11. At $\bullet^4$, do not penalise candidates for who reduce powers of 0. For example writing 0 in place of 0^4. Similarly, do not penalise candidates writing 0 in place of $6(0)$. However, candidates who write 0^3 in place of 0^4 or $2(0)$ in place of $6(0)$ do not gain $\bullet^4$.

Commonly Observed Responses:

Candidate A - switched limits

$$\int_2^0\left(\left(6+4x-2x^2\right)-\left(x^3-6x^2+11x\right)\right)dx \qquad \bullet^2\ ✓$$

$$=6x-\frac{7}{2}x^2+\frac{4}{3}x^3-\frac{1}{4}x^4 \qquad \bullet^3\ ✓$$

$$=0-\left(6(2)-\frac{7}{2}(2)^2+\frac{4}{3}(2)^3-\frac{1}{4}(2)^4\right) \qquad \bullet^4\ ✓$$

$$=-\frac{14}{3}$$

$$=\frac{14}{3} \qquad \bullet^1\ ✗\quad \bullet^5\ ✗$$

Candidate B - 'lower' - 'upper'

$$\int_0^2\left(\left(x^3-6x^2+11x\right)-\left(6+4x-2x^2\right)\right)dx \qquad \bullet^2\ ✓$$

$$\int_0^2 x^3-4x^2+7x-6\,dx$$

$$=\frac{1}{4}x^4-\frac{4}{3}x^3+\frac{7}{2}x^2-6x \qquad \bullet^3\ ✓$$

$$\left(\frac{1}{4}(2)^4-\frac{4}{3}(2)^3+\frac{7}{2}(2)^2-6(2)\right)-(0) \qquad \bullet^4\ ✓$$

$$=-\frac{14}{3}$$

$$\therefore \text{Area}=\frac{14}{3} \qquad \bullet^1\ ✓\quad \bullet^5\ ✓$$

Question	Generic scheme	Illustrative scheme	Max mark
7. (continued)			

Candidate C - missing brackets $\int_0^2 6+4x-2x^2-x^3-6x^2+11x\,dx$ $\int_0^2 6-7x+4x^2-x^3\,dx$ •1 ✓ •2 ✓	**Candidate D - missing brackets** $\int_0^2 6+4x-2x^2-x^3-6x^2+11x\,dx$ •1 ✗ •2 ✓1 $\int_0^2 6+15x-8x^2-x^3\,dx$ $6x+\frac{15}{2}x^2-\frac{8}{3}x^3-\frac{1}{4}x^4$ •3 ✓1 $\left(6(2)+\frac{15}{2}(2)^2-\frac{8}{3}(2)^3-\frac{1}{4}(2)^4\right)-(0)$ •4 ✓1 $\frac{50}{3}$ •5 ✓1
Candidate E - 'upper' + 'lower' $\int_0^2\left(\left(6+4x-2x^2\right)+\left(x^3-6x^2+11x\right)\right)dx$ •1 ✗ •2 ✓1 $6x+\frac{15}{2}x^2-\frac{8}{3}x^3+\frac{1}{4}x^4$ •3 ✓1 $\left(6(2)+\frac{15}{2}(2)^2-\frac{8}{3}(2)^3+\frac{1}{4}(2)^4\right)-0$ •4 ✓1 $\frac{74}{3}$ •5 ✓1	**Candidate F - incorrect substitution** $\int_0^2\left(\left(6+4x-2x^2\right)-\left(x^3-6x^2+11x\right)\right)dx$ •1 ✓ •2 ✓ $\left(6x+2x^2-\frac{2}{3}x^3\right)-\left(\frac{1}{4}x^4-2x^3+\frac{11}{2}x^2\right)$ •3 ✓ $\left(6(2)+2(2)^2-\frac{2}{3}(2)^3\right)-\left(\frac{1}{4}(0)^4-2(0)^3+\frac{11}{2}(0)^2\right)$ •4 ✗ $\frac{44}{3}$ •5 ✓2

Question			Generic scheme	Illustrative scheme	Max mark
8.	(a)		$\bullet^1$ interpret notation	$\bullet^1$ $f(x+1)$ or $2g(x)^2-18$	**2**
			$\bullet^2$ state expression for $f(g(x))$	$\bullet^2$ $2(x+1)^2-18$	

Notes:

1. For $2(x+1)^2-18$ without working, award both $\bullet^1$ and $\bullet^2$.

Commonly Observed Responses:

Candidate A - $g(f(x))$	**Candidate B - beware of two "attempts"**
$2x^2-17$ $\qquad \bullet^1 \times \ \bullet^2 \checkmark_1$	$f(g(x))=2x^2-18$ $\qquad \bullet^1 \times \ \bullet^2 \times$ $f(x+1)=2(x+1)^2-18$

Question			Generic scheme	Illustrative scheme	Max mark
	(b)		$\bullet^3$ apply condition	$\bullet^3$ $2(x+1)^2-18=0$	**2**
			$\bullet^4$ state values of x	$\bullet^4$ -4 and 2	

Notes:

2. Working at $\bullet^3$ must be consistent with working at $\bullet^2$.
3. Accept $2(x+1)^2-18\neq 0$ for $\bullet^3$ only when $x=-4$ and $x=2$ are stated explicitly at $\bullet^4$. See Candidate H
4. $\bullet^4$ is only available for finding the roots of a quadratic.
5. For subsequent incorrect working, the final mark is not available. For example $-4<x<2$.

Commonly Observed Responses:

Candidate C - expanding brackets in (a)	**Candidate D - expanding brackets in (a)**
Part (a) $f(g(x))=2(x+1)^2-18$ $\qquad \bullet^1 \checkmark \ \bullet^2 \checkmark$ $f(g(x))=2x^2+4x-16$ **Part (b)** $2x^2+4x-16=0$ $\qquad \bullet^3 \checkmark$ $x=-4$ and $x=2$ $\qquad \bullet^4 \checkmark$	**Part (a)** $f(g(x))=2(x+1)^2-18$ $\qquad \bullet^1 \checkmark \ \bullet^2 \checkmark$ $f(g(x))=2x^2-16$ Part (b) $2x^2-16=0$ $\qquad \bullet^3 \times$ $x=\pm\sqrt{8}$ $\qquad \bullet^4 \checkmark_1$
Candidate E - $g(f(x))$ **Part (a)** $f(g(x))=2x^2-17$ $\qquad \bullet^1 \times \ \bullet^2 \checkmark_1$ Part (b) $2x^2-17=0$ $\qquad \bullet^3 \checkmark_1$ $x=\pm\sqrt{\frac{17}{2}}$ $\qquad \bullet^4 \checkmark_1$	**Candidate F - equivalent condition** $2(x+1)^2=18$ $\qquad \bullet^3 \checkmark$
Candidate G - use of $\neq$ $2(x+1)^2-18\neq 0$ $\qquad \bullet^3 \times$ $x\neq -4,\ x\neq 2$ $\qquad \bullet^4 \checkmark_1$	**Candidate H - use of $\neq$** $2(x+1)^2-18\neq 0$ $x\neq -4,\ x\neq 2$ $x=-4,\ x=2$ $\qquad \bullet^3 \checkmark$ $\bullet^4 \checkmark$

Question			Generic scheme	Illustrative scheme	Max mark
9.	(a)		•¹ differentiate two non-constant terms •² complete derivative and equate to 0 •³ find x-coordinates •⁴ find y-coordinates	•¹ eg $x^2 - 2x$ •² $x^2 - 2x - 3 = 0$ •³ •⁴ •³ -1, 3 •⁴ $\frac{8}{3}$, -8	4

Notes:

1. For a numerical approach, award 0/4.
2. •² is only available if '$=0$' appears at the •² stage or in working leading to •³. However, see Candidate A.
3. •³ is only available for solving a quadratic equation.
4. •³ and •⁴ may be awarded vertically.

Commonly Observed Responses:

Candidate A

Stationary points when $\frac{dy}{dx} = 0$

$\frac{dy}{dx} = x^2 - 2x - 3$ •¹ ✓ •² ✓

$\frac{dy}{dx} = (x+1)(x-3)$

$x = -1, 3$ •³ ✓

$y = \frac{8}{3}, -8$ •⁴ ✓

Candidate B - derivative never equated to 0

$x^2 - 2x - 3$ •¹ ✓ •² ^

$(x+1)(x-3)$

$x = -1, 3$ •³ ✓₁

$y = \frac{8}{3}, -8$ •⁴ ✓

Question			Generic scheme	Illustrative scheme	Max mark
	(b)		•⁵ evaluate y at $x = 6$ •⁶ state greatest and least values	•⁵ 19 •⁶ greatest = 19 and least = -8	2

Notes:

5. 'Greatest $(6,19)$; least $(3,-8)$' does not gain •⁶.
6. Where $x = -1$ was not identified as a stationary point in part (a), y must also be evaluated at $x = -1$ to gain •⁶.
7. •⁶ is not available for using y at a value of x, obtained at •³ stage, which lies outwith the interval $-1 \leq x \leq 6$.
8. •⁶ is only available where candidates have attempted to evaluate y at $x = 6$.

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
10.	(a)		•1 state centre •2 calculate radius	•1 $(-9,1)$ •2 $\sqrt{90}$ or $3\sqrt{10}$ or $9.48\ldots$	**2**

Notes:

1. Accept $x=-9,\ y=1$ for •1.
2. Do not accept '$g=-9,\ f=1$' or '$-9,1$' for •1.
3. Do not penalise candidates who treat negative signs with a lack of rigour when calculating the radius. For example accept $\sqrt{9^2+-1^2+8}=\sqrt{90}$ or $\sqrt{9^2+1^2+8}=\sqrt{90}$ or $\sqrt{-9^2+1^2+8}=\sqrt{90}$ for •2. However, do not accept $\sqrt{9^2-1^2+8}=\sqrt{90}$ for •2.

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
	(b)		•3 determine the distance between the centres and subtract to find a numerical expression for the radius of C_2 •4 determine equation of C_2	•3 eg $\sqrt{90}-\sqrt{10}$ •4 $(x+6)^2+y^2=40$	**2**

Notes:

4. Do not penalise the use of decimals.
5. The distance between the centres, and the radius of C_2 must be consistent with the sizes of the circles in the original diagram ($d<r_{C_2}<r_{C_1}$).
6. Where a candidate uses an incorrect radius without supporting working, •4 is not available.

Commonly Observed Responses:

Candidate A - follow-through marking

Part (a)

$r=\sqrt{74}$ •2 ✗

Part (b)

$d=\sqrt{10}$

radius $=\sqrt{74}-\sqrt{10}$ •3 ✓1

$(x+6)^2+y^2=5.44\ldots^2$

$(x+6)^2+y^2=29.59\ldots$ (or $84-4\sqrt{185}$) •4 ✓1

Candidate B - using line through centres

Equation of radius: $3y=-x-6$

$(-3y-6)^2+y^2+18(-3y-6)-2y-8=0$

$10y^2-20y-80=0$

$\cancel{y=4}$ $y=-2$

$\cancel{x=-18}$ $x=0$

Radius = distance between $(-6,0)$ and $(0,-2)$

Radius = $\sqrt{40}$ •3 ✓

$(x+6)^2+y^2=40$ •4 ✓

Question			Generic scheme	Illustrative scheme	Max mark
11.	(a)		•1 state number of vehicles	•1 6.8 million	1

Notes:

1. Accept 6.8 or $N = 6.8$ million for •1.

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
	(b)		•2 substitute for N and t	•2 $125 = 6.8e^{10k}$ stated or implied by •3	4
			•3 process equation	•3 $\frac{125}{6.8} = e^{10k}$	
			•4 express in logarithmic form	•4 $\log_e\left(\frac{125}{6.8}\right) = 10k$	
			•5 solve for k	•5 0.2911...	

Notes:

2. Accept answers which round to 0.29.
3. Do not penalise rounding or transcription errors (which are correct to 2 significant figures) in intermediate calculations.
4. •3 may be assumed by •4.
5. Any base may be used at •4 stage. See Candidate A.
6. At •4 all exponentials must be processed.
7. Accept $\log_e \frac{125}{6.8} = 10k \log_e e$ for •4.
8. The calculation at •5 must follow from the valid use of exponentials and logarithms at •3 and •4.
9. For candidates with no working, or who adopt an iterative approach to arrive at $k = 0.29$, award 1/4. However, if, in the iterations N is calculated for $k = 0.295$ and $k = 0.285$, then award 4/4.

Commonly Observed Responses:

Candidate A - use of alternative base

$125 = 6.8e^{10k}$ •2 ✓

$\frac{125}{6.8} = e^{10k}$ •3 ✓

$\log_{10}\left(\frac{125}{6.8}\right) = 10k \log_{10} e$ •4 ✓

$k = 0.2911...$ •5 ✓

Candidate B - missing lines of working

$125 = 6.8e^{10k}$ •2 ✓

$k = 0.2911...$ •3 ^ •4 ^ •5 ✓

Candidate C - errors in substitution

$125\,000\,000 = 6.8e^{10k}$ •2 ✗

$\frac{125\,000\,000}{6.8} = e^{10k}$ •3 $✓_1$

$16.726... = 10k$ •4 $✓_1$

$k = 1.6726...$ •5 $✓_1$

Question			Generic scheme	Illustrative scheme	Max mark
12.			•1 substitute appropriate double angle formula •2 factorise •3 solve for $\tan x°$ •4 solve $\tan x° = 4$ •5 solve $\sin x° = 0$	•1 $2(2\sin x°\cos x°) - \sin^2 x° (= 0)$ •2 $\sin x°(4\cos x° - \sin x°) = 0$ •3 $\tan x° = 4$ (since $x = 90, 270$ are not solutions) •4 •5 •4 76, 256 •5 0, 180	5

Notes:

1. •1 is still available to candidates who correctly substitute for $\sin^2 x$ in addition to $\sin 2x$.
2. Substituting $2\sin A\cos A$ for $\sin 2x°$ at the •1 stage should be treated as bad form provided the equation is written in terms of x at the •2 stage. Otherwise, •1 is not available.
3. '$=0$' must appear by the •2 stage for •2 to be awarded.
4. Award •2 for '$S(4C - S) = 0$' only where $\sin x° = 0$ and $4\cos x° - \sin x° = 0$ appear.
5. Do not penalise the omission of degree signs.
6. At •3 stage, candidates are not required to check that 90 and 270 are not solutions before dividing by $\cos x°$. Where candidates have divided by $\sin x$ at the •2 stage without considering $\sin x = 0$, •3 and •4 are still available.
7. At •3 stage, candidates may use the wave function and arrive at $\sqrt{17}\cos(x + 14)° = 0$, or an equivalent wave form, instead of $\tan x° = 4$.
8. •4 is only available where the working at the •3 stage is of equivalent difficulty to reaching $\tan x° = 4$.
9. •5 is not available where $\sin x = 0$ comes from an invalid strategy.
10. For candidates who work only in radians, •5 is not available.
11. •4 and •5 may be awarded vertically. See also Candidate B.
12. Do not penalise solutions outwith $0 \le x < 360$.

Commonly Observed Responses:

Candidate A - working in radians

⋮ •1 ✓ •2 ✓

$\tan x° = 4$ •3 ✓

1.326, 4.468 •4 ✓1

$0, \pi$ •5 ✓2

Candidate B - partial solutions

$2(2\sin x°\cos x°) - \sin^2 x° = 0$ •1 ✓

$\sin x°(4\cos x° - \sin x°) = 0$ •2 ✓

$\sin x° = 0$

$x = 180$

$\tan x° = 4$ •3 ✓

$x = 76$ •4 ✓

•5 ^

Question			Generic scheme	Illustrative scheme	Max mark
13.			•1 state repeated factor	•1 $(x-3)^2(\ldots)(\ldots)$	3
			•2 state non-repeated linear factors	•2 $(\ldots)^2(x+1)(x-5)$	
			•3 calculate k and express in required form	•3 $f(x)=\frac{1}{5}(x-3)^2(x+1)(x-5)$	

Notes:

1. Do not penalise the omission of $f(x)=$ or the inclusion of $y=$.
2. Accept $f(x)=\frac{1}{5}(x+-3)^2(x+1)(x+-5)$ for •3.

Commonly Observed Responses:

Candidate A - incorrect signs $f(x)=k(x+3)^2(x-1)(x+5)$ •1 ✗ •2 $\checkmark_1$ $f(x)=\frac{1}{5}(x+3)^2(x-1)(x+5)$ •3 $\checkmark_1$	**Candidate B - incorrect repeated root** $f(x)=k(x+1)^2(x-3)(x-5)$ •1 ✗ •2 $\checkmark_1$ $f(x)=-\frac{3}{5}(x+1)^2(x-3)(x-5)$ •3 $\checkmark_1$
Candidate C - incorrect repeated root $f(x)=k(x-5)^2(x+1)(x-3)$ •1 ✗ •2 $\checkmark_1$ $f(x)=\frac{3}{25}(x-5)^2(x+1)(x-3)$ •3 $\checkmark_1$	**Candidate D - incorrect signs and repeated root** $f(x)=k(x+5)^2(x-1)(x+3)$ •1 ✗ •2 ✗ $f(x)=\frac{3}{25}(x+5)^2(x-1)(x+3)$ •3 $\checkmark_1$
Candidate E - incorrect signs and repeated root $f(x)=k(x-1)^2(x+5)(x+3)$ •1 ✗ •2 ✗ $f(x)=-\frac{3}{5}(x-1)^2(x+5)(x+3)$ •3 $\checkmark_1$	**Candidate F - use of *a*, *b* and *c*** $a=-3$ •1 ✓ $b=1,\ c=-5$ (or $b=-5,\ c=1$) •2 ✓ $k=\frac{1}{5}$ •3 ^

[END OF MARKING INSTRUCTIONS]